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Topic 12

Digital Basics

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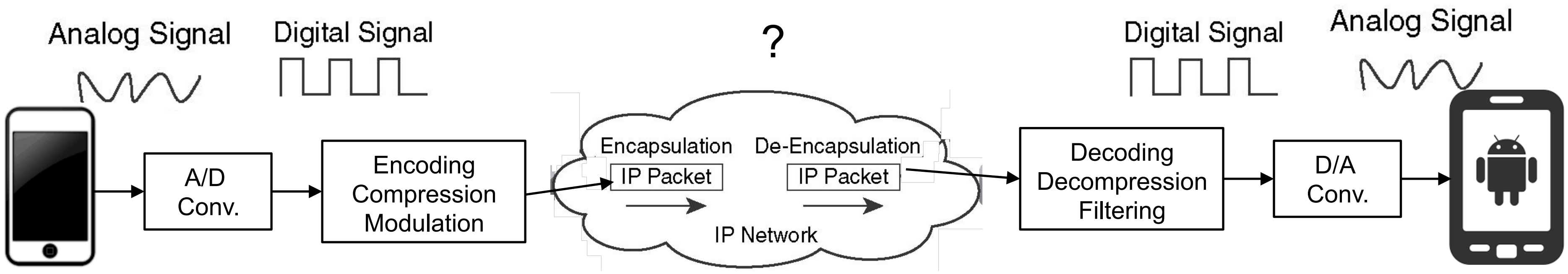
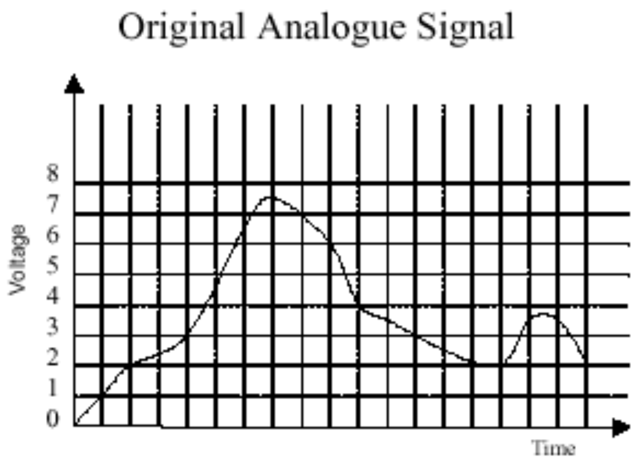
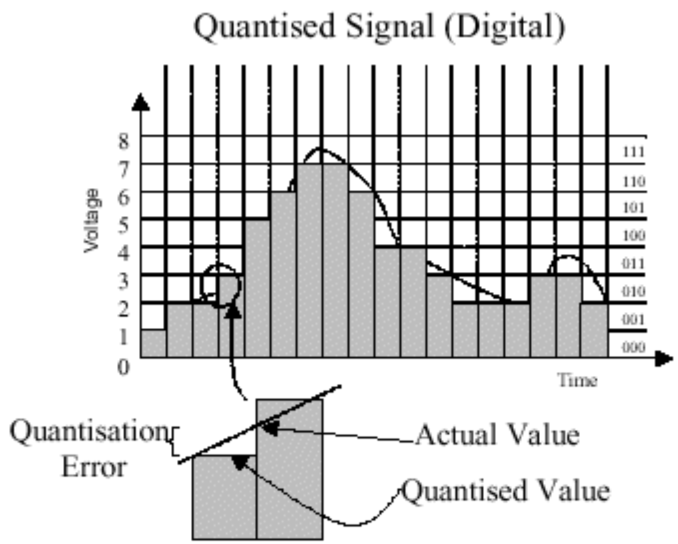
Learning outcomes on digital electronics

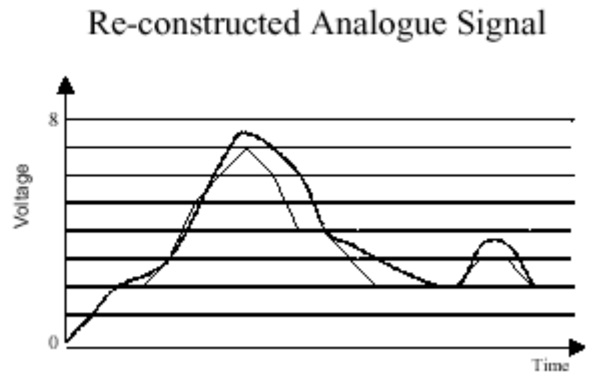
- Understand the **formalism of logic** and able to **analyse logical processes**.
- Implement **simple logical operations** using combinational logic circuits.
- Understand common forms of **number representation** in digital electronic circuits and to be able to convert between different representations.
- Understand the logical operation of simple **arithmetic** and other **MSI circuits (Medium Scale Integrated Circuits)**
- Understand the concepts of **sequential circuits** enabling you to analyse sequential systems in terms of **state machines** and **counters**.
- Understand how **digital storage** (e.g. memory) works and how its **content is accessed**.
- Understand the basics of **microprocessors** and **microcontrollers**.
- Able to **integrate hardware and software** together in a simple electronic system.
- Interface electronic circuits to the physical world and **process analogue signals on microcontroller** systems in digital form.

Analogue vs Digital









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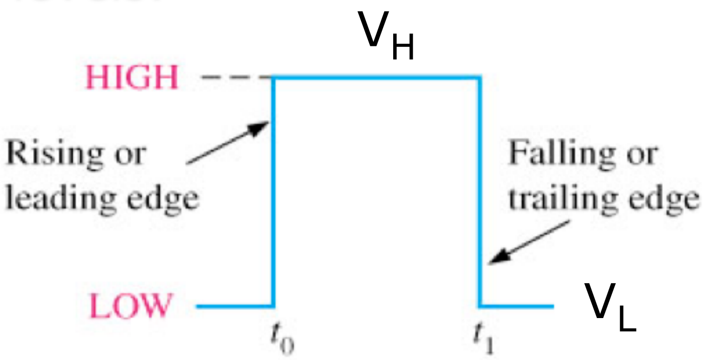
- Most physical phenomena are in the analogue domain.
- Most modern electronics systems operate in the digital domain.
- Analogue-to-Digital (A/D) converters, and Digital-to-Analogue (D/A) converters links the two worlds together.

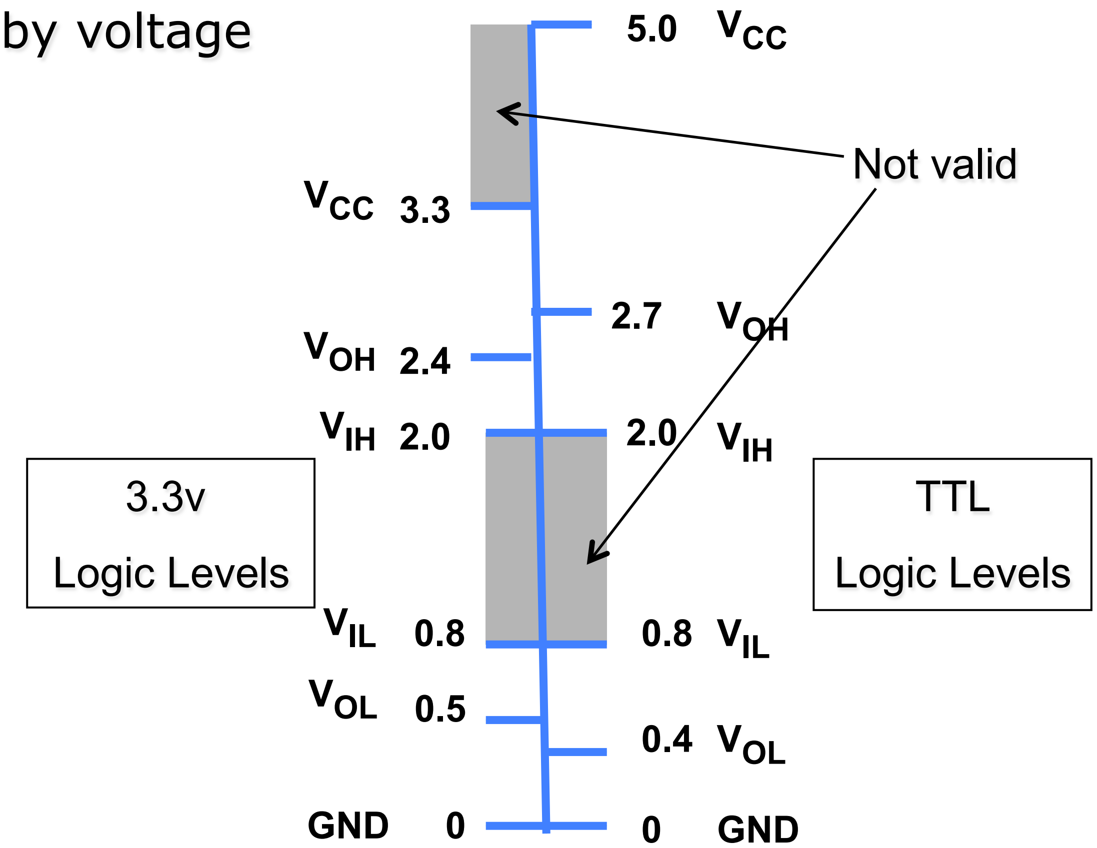
Binary Digits, Logic Levels

- The conventional numbering system uses ten digits: **0** to **9**.
- The binary numbering system uses just two digits: **0** and **1.**
- They can also be called LOW and HIGH, FALSE and TRUE, or 0 and 1.

Binary values are also represented by voltage levels.

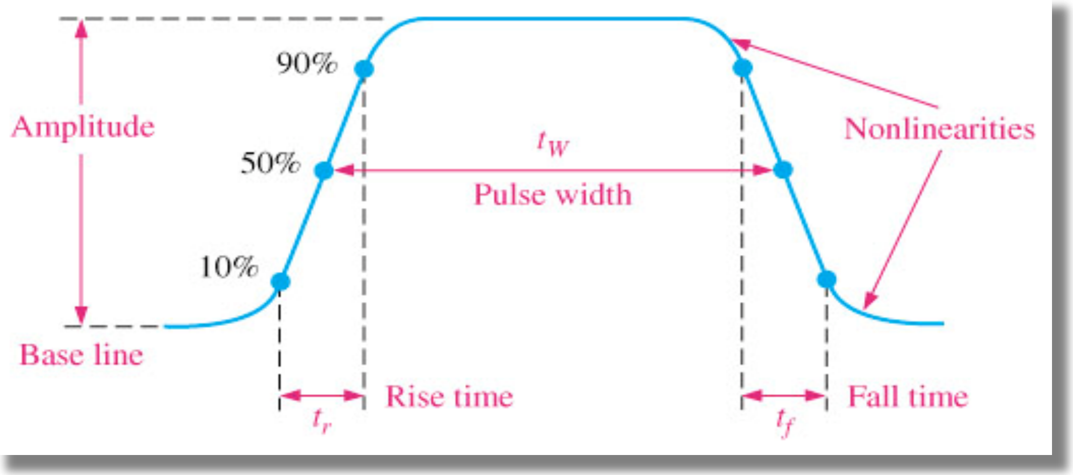


- VCC – Logic supply voltage level
- VOH – Logic high output level
- VIH – Logic high input level
- VIL – Logic low input level
- VOL – Logic low output level

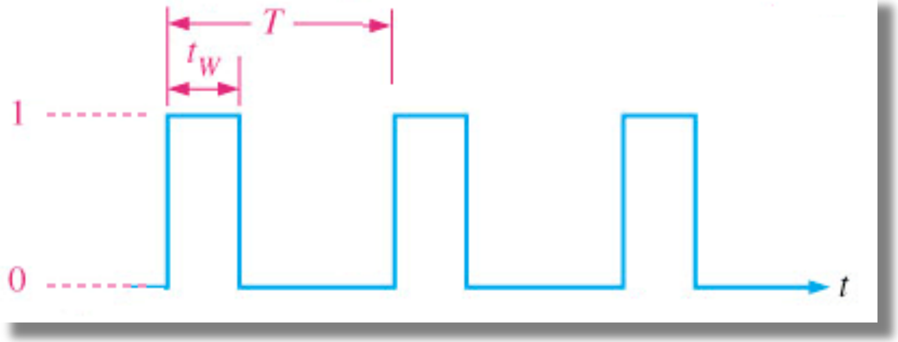


Digital Waveforms

- Major parts of a digital pulse
- Base line
- Amplitude
- Rise time (t_r)
- Pulse width (t_w)
- Fall time (t_f)
- Period (T)
- Frequency (f)



$f = 1/T$ in Hz



The duty cycle of a binary waveform is defined as:

Duty Cycle = $(t_w / T) \times 100\ \%$

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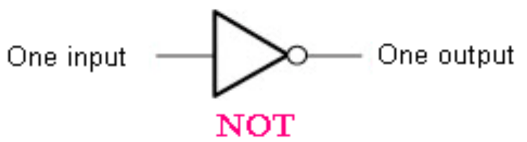
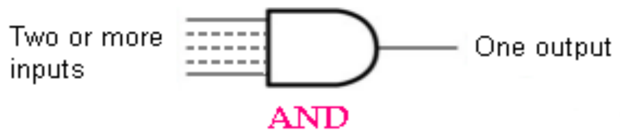
Basic Logic Operations

There are only three basic logic operations:



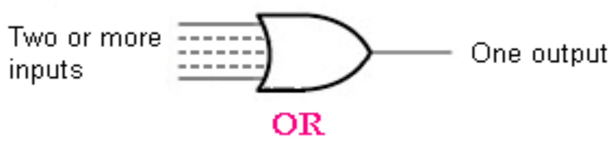
NOT gate

Input	Output
0	1
1	0



AND gate

Input A	Input B	Output
0	0	0
1	0	0
0	1	0
1	1	1



OR gate

Input A	Input B	Output
0	0	0
1	0	1
0	1	1
1	1	1

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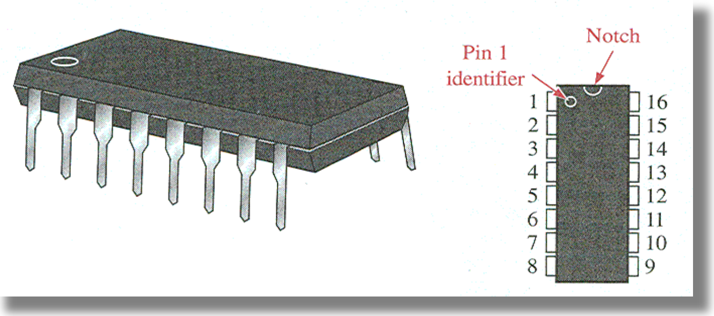
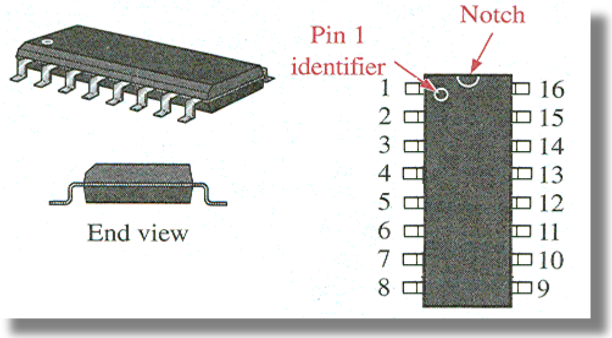
Common integrated circuit packages

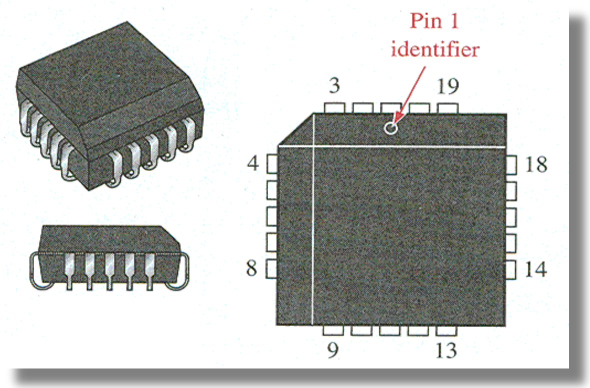
Dual in-line package (DIP)



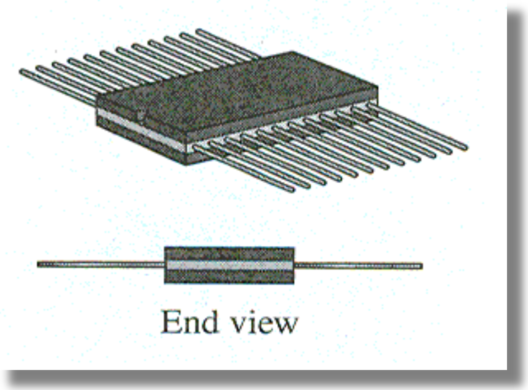
Small-outline IC (SOIC)



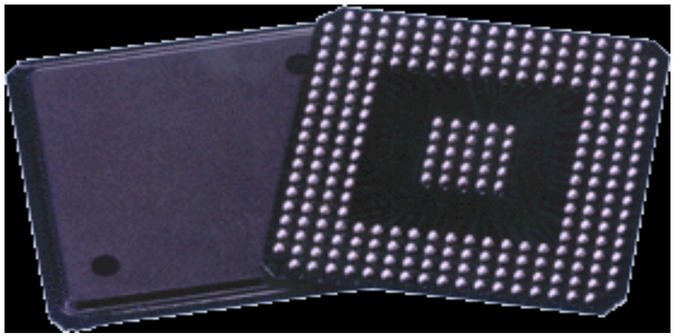
Plastic-leaded chip carrier (PLCC)



Flat pack (FP)



Ball Grid Array (BGA)

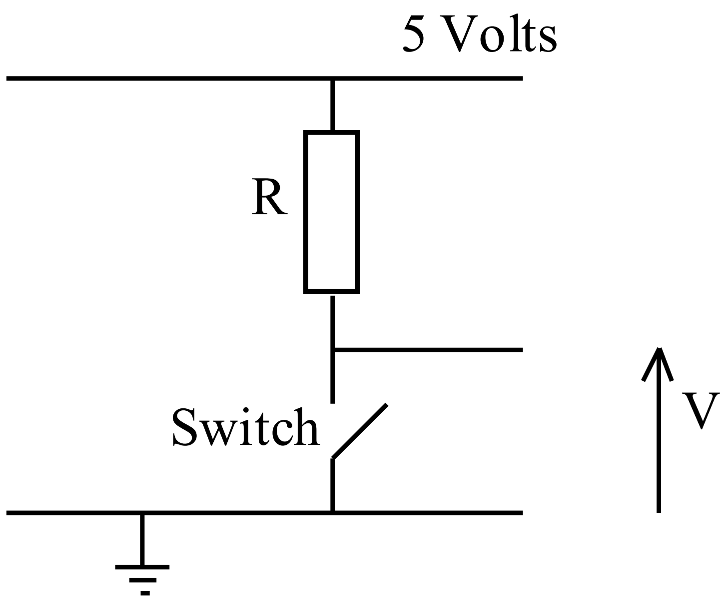


What do we mean by data?

- Many definitions are possible depending on context
- We will say that:
  - data is a physical representation of information
- Data can be stored
  - e.g. computer disk, memory chips
- Data can be transmitted
  - e.g. internet
- Data can be processed
  - e.g. inside a microprocessor

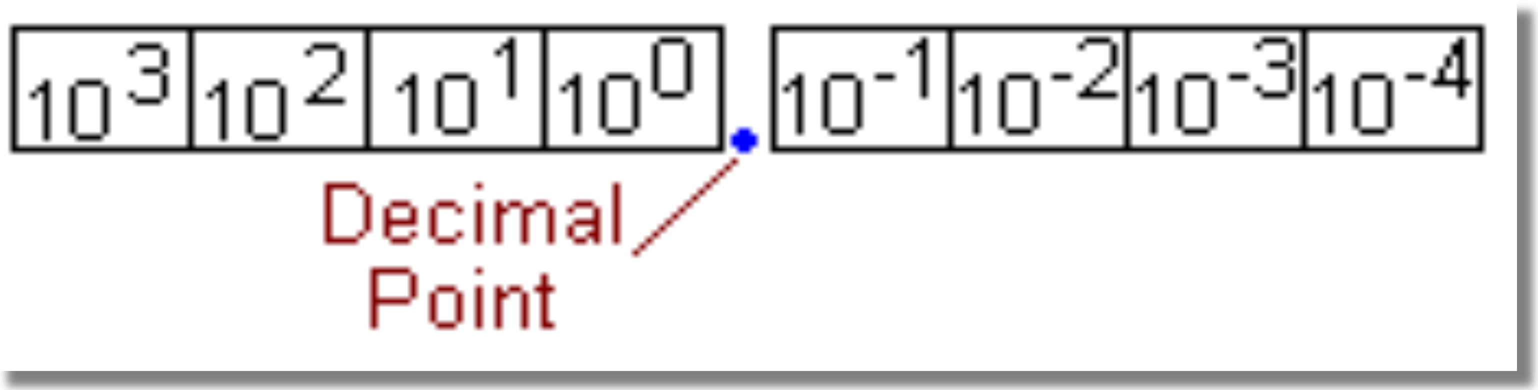
Electronic Representation of Data

- Information can be very complicated
  - e.g.:
    - Numbers
    - Sounds
    - Pictures
    - Codes
  - We need a simple electronic representation
- What can we do with electronics?
  - Set up voltages and currents
  - Change the voltages and currents
- A useful device is a switch
  - Switch Closed: V = 0 Volts
  - Switch Open: V = 5 Volts



Decimal Numbers

- The decimal number system has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9
- The decimal numbering system has a base of 10 with each position weighted by a factor of 10:



Binary Numbers

- The binary number system has two digits:
  0 and 1

- The binary numbering system has a base of 2 with each position weighted by a factor of 2:

POSITIVE POWERS OF TWO (WHOLE NUMBERS)									NEGATIVE POWERS OF TWO (FRACTIONAL NUMBER)					
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
256	128	64	32	16	8	4	2	1	1/2	1/4	1/8	1/16	1/32	1/64
									0.5	0.25	0.125	0.0625	0.03125	0.015625

Binary Number System

- Uses 2 symbols by our previous rule
  - 0 and 1
- Example: 10011 in binary is

  $1 \times 2^4 + 1 \times 2^1 + 1 \times 2^0 = 19$

2^4	2^3	2^2	2^1	2^0
1	0	0	1	1

- Binary is the base 2 number system
- Most common in digital electronics

Integer and Fractional Parts

- Binary numbers can contain fractional parts as well as integer parts

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
1	0	0	1	1	0	1	1

Binary Point

$\Rightarrow (19.375)_{10}$

- This 8-bit number is in Q3 format
  - 3 bits after the binary point
- How could 19.376 best be represented using an 8-bit binary number?
  - Quantization error

Conversion: decimal to binary (Method 1)

- The decimal number is simply expressed as a sum of powers of 2, and then 1s and 0s are written in the appropriate bit positions.

$$
\begin{aligned}
50_{10} &= 32 + 18 \\
&= 32 + 16 + 2 \\
&= 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^1 \\
50_{10} &= 110010_2
\end{aligned}
$$

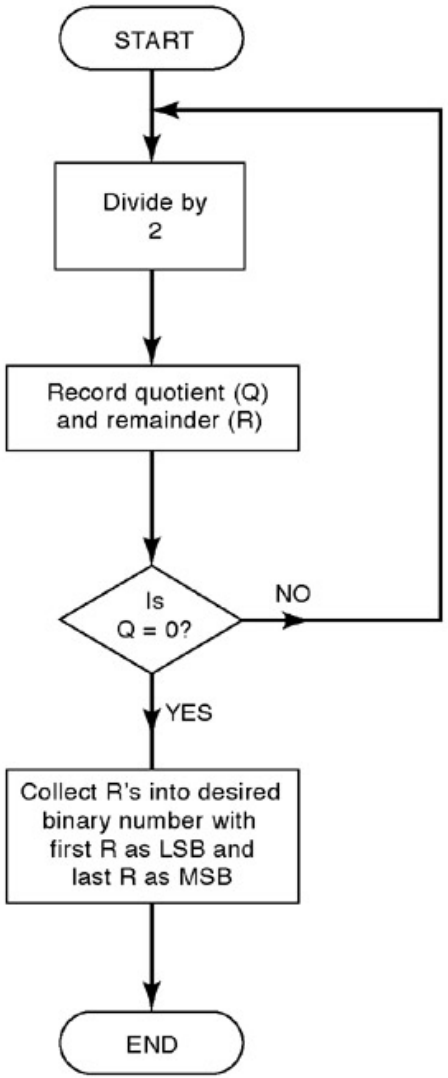
$$
\begin{aligned}
346_{10} &= 256 + 90 \\
&= 256 + 64 + 26 \\
&= 256 + 64 + 16 + 10 \\
&= 256 + 64 + 16 + 8 + 2 \\
&= 1 \times 2^8 + 1 \times 2^6 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^1 \\
346_{10} &= 101011010_2
\end{aligned}
$$

Conversion: decimal to binary (method 2)

- Repeated division

		quotient	remainder	
50/2	=	25	0	LSB
25/2	=	12	1	
12/2	=	6	0	
6/2	=	3	0	
3/2	=	1	1	
1/2	=	0	1	MSB

$$50_{10}=110010_2$$



Conversion: binary to decimal

- The simplest way is to represent an n-bit binary number as

  $a_n \times 2^{n-1} + \ldots + a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0$

- The conversion can be done by substituting the a's with the given bits then multiplying and adding:
  - eg: Convert $(1101)_2$ into decimal
  - $1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (13)_{10}$

- Other algorithms can be used as alternatives if you prefer

Binary Addition

- First recall decimal addition

	1	1	1	
A	1	2	3	4
+ B		9	8	7
Sum	2	2	2	1

- In binary addition we follow the same pattern but
  - 0 + 0 = 0 carry-out 0
  - 0 + 1 = 1 carry-out 0
  - 1 + 0 = 1 carry-out 0
  - 1 + 1 = 0 carry-out 1
  - 1 + 1 + carry-in = 1 carry-out 1

		1		
A	0	1	1	1
+ B	0	1	1	0
Sum	1	1	0	1

- Note that we need to consider 3 inputs per bit of binary number
  - A, B and carry-in
- Each bit of binary addition generates 2 outputs
  - sum and carry-out

Hexadecimal Numbers

- Decimal, binary, and hexadecimal numbers

DECIMAL	BINARY	HEXADECIMAL
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Hexadecimal Numbers conversions

- ## Binary-to-hexadecimal conversion
  1. Break the binary number into 4-bit groups
  2. Replace each group with the hexadecimal equivalent

- ## Hexadecimal-to-decimal conversion
  1. Convert the hexadecimal to groups of 4-bit binary
  2. Convert the binary to decimal

- ## Decimal-to-hexadecimal conversion
  – Repeated division by 16

Binary Coded Decimal (BCD)

- Use 4-bit binary to represent one decimal digit
- Easy conversion
- Wasting bits (4-bits can represent 16 different values, but only 10 values are used)
- Used extensively in financial applications

DECIMAL DIGIT	0	1	2	3	4	5	6	7	8	9
BCD	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

Binary Coded Decimal (BCD)

- Convert 0110100000111001(BCD) to its decimal equivalent.

  0110 1000 0011 1001

  6 8 3 9

- Convert the BCD number 011111000001 to its decimal equivalent.

  0111 1100 0001

  7 ↑ 1

  The forbidden code group indicated an error

Summary – binary, hexadecimal and BCD

Decimal	Binary	Octal	Hexadecimal	BCD
0	0	0	0	0000
1	1	1	1	0001
2	10	2	2	0010
3	11	3	3	0011
4	100	4	4	0100
5	101	5	5	0101
6	110	6	6	0110
7	111	7	7	0111
8	1000	10	8	1000
9	1001	11	9	1001
10	1010	12	A	0001 0000
11	1011	13	B	0001 0001
12	1100	14	C	0001 0010
13	1101	15	D	0001 0011
14	1110	16	E	0001 0100
15	1111	17	F	0001 0101

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ASCII code

Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char
0	0	*[NULL]*	32	20	*[SPACE]*	64	40	@	96	60	`
1	1	*[START OF HEADING]*	33	21	!	65	41	A	97	61	a
2	2	*[START OF TEXT]*	34	22	"	66	42	B	98	62	b
3	3	*[END OF TEXT]*	35	23	#	67	43	C	99	63	c
4	4	*[END OF TRANSMISSION]*	36	24	$	68	44	D	100	64	d
5	5	*[ENQUIRY]*	37	25	%	69	45	E	101	65	e
6	6	*[ACKNOWLEDGE]*	38	26	&	70	46	F	102	66	f
7	7	*[BELL]*	39	27	'	71	47	G	103	67	g
8	8	*[BACKSPACE]*	40	28	(	72	48	H	104	68	h
9	9	*[HORIZONTAL TAB]*	41	29	)	73	49	I	105	69	i
10	A	*[LINE FEED]*	42	2A	*	74	4A	J	106	6A	j
11	B	*[VERTICAL TAB]*	43	2B	+	75	4B	K	107	6B	k
12	C	*[FORM FEED]*	44	2C	,	76	4C	L	108	6C	l
13	D	*[CARRIAGE RETURN]*	45	2D	-	77	4D	M	109	6D	m
14	E	*[SHIFT OUT]*	46	2E	.	78	4E	N	110	6E	n
15	F	*[SHIFT IN]*	47	2F	/	79	4F	O	111	6F	o
16	10	*[DATA LINK ESCAPE]*	48	30	0	80	50	P	112	70	p
17	11	*[DEVICE CONTROL 1]*	49	31	1	81	51	Q	113	71	q
18	12	*[DEVICE CONTROL 2]*	50	32	2	82	52	R	114	72	r
19	13	*[DEVICE CONTROL 3]*	51	33	3	83	53	S	115	73	s
20	14	*[DEVICE CONTROL 4]*	52	34	4	84	54	T	116	74	t
21	15	*[NEGATIVE ACKNOWLEDGE]*	53	35	5	85	55	U	117	75	u
22	16	*[SYNCHRONOUS IDLE]*	54	36	6	86	56	V	118	76	v
23	17	*[ENG OF TRANS. BLOCK]*	55	37	7	87	57	W	119	77	w
24	18	*[CANCEL]*	56	38	8	88	58	X	120	78	x
25	19	*[END OF MEDIUM]*	57	39	9	89	59	Y	121	79	y
26	1A	*[SUBSTITUTE]*	58	3A	:	90	5A	Z	122	7A	z
27	1B	*[ESCAPE]*	59	3B	;	91	5B	[	123	7B	{
28	1C	*[FILE SEPARATOR]*	60	3C	<	92	5C	\	124	7C	|
29	1D	*[GROUP SEPARATOR]*	61	3D	=	93	5D	]	125	7D	}
30	1E	*[RECORD SEPARATOR]*	62	3E	>	94	5E	^	126	7E	~
31	1F	*[UNIT SEPARATOR]*	63	3F	?	95	5F	_	127	7F	*[DEL]*